Impacts of MHD boundary layer flow and melting heat transfer with chemical reaction and radiation

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Abstract

The present investigation deals with the study of the effects of chemical reaction and thermal radiation on the steady MHD boundary layer flow and heat transfer melting of Williamson nanofluid embedded in porous medium over a horizontal linearly stretching sheet under the influence of heat source and viscous dissipation. The governing momentum boundary layer and thermal boundary layer equations with the boundary conditions are transformed into a system of nonlinear ordinary differential equations which are then solved numerically by using the Runge–Kutta–Fehlberg method. Numerical results for the dimensionless velocity, temperature and concentration profiles as well as for the skin friction factor, Nusselt number and Sherwood number are elucidated for different values of the pertinent parameters. Comparison with existing literature is shown and it found to be in good agreement.

Keywords: ***Heat transfer melting parameter; MHD; Boundary layer; Chemical reaction; Viscous dissipation; nanofluids.***

Introduction

Nanofluids are attracting a great deal of interest due to their enormous potential with respect to enhanced heat transfer. The term “nanofluid” describes a liquid suspension composed of tiny particles of diameter less than 100 nm. It defines an important class of fluids, which has a distinctive ability to improve the thermal properties of fluids. The effective thermal conductivity of the base fluid is appreciably enhanced as a consequence of the addition of small amount of nanoparticles according to the experimental verification by Choi [1]. Kleinstreuer and Feng [2] derived the experimental and theoretical studies of nanofluid thermal conductivity enhancement. The comparison of nanofluid thermal conductivity and heat transfer enhancements was studied by Yu et al. [3]. Eapen et al. [4] derived the classical nature of thermal conduction in nanofluids. Chitra and Sendhilnathan [5] investigated on the thermal studies of nanofluids related to their applications. The effects of Brownian motion and thermophoresis added into nanofluid model was first derived by Buongiorno [6].

Magneto hydrodynamic boundary-layer flow of nanofluid and heat transfer has received a lot of attention in the field of several industrial, scientific, and engineering applications in recent years. Mabood et al. [8] proposed a numerical study on MHD boundary layer flow of Nano fluids over a nonlinear stretching sheet with heat transfer effect. Analytical solution of free convective flow of a nanofluid over a stretching sheet in the presence of magnetic field was carried out by Hamad [9]. Ibrahim et al. [10] studied the MHD stagnation point flow and heat transfer due to nanofluid towards a stretching sheet. MHD boundary layer flow of a nanofluid past a wedge was illustrated by Srinivasacharya et al. [11].

Motivated by some of the researchers mentioned above and its applications in various fields of science and technology, it is of interest to discuss and analyze the thermal radiation and chemical reaction effects on the MHD boundary layer flow and melting heat transfer of Williamson nanofluid in a porous medium under the influence of viscous dissipation and heat generation. In the present study, the governing equations are solved numerically by Runge-Kutta –Fehlberg 45 method along with shooting technique.

Mathematical formulation

A steady two-dimensional flow of viscous incompressible Williamson nan fluid over a stretching surface in a porous medium is considered. The plate is stretched along x-axis with a velocity ax, where $a > 0$ is stretching parameter. U_w, T_w and C_w are the velocity, temperature and nanoparticle concentration near surface. Let the temperature of the melting surface as T_m, and temperature in the free-stream condition as T_∞, where $T_\infty > T_m$. The viscous dissipation and heat source or absorption are added into the flow. For the present paper, the basic equations of conservation of mass, momentum, energy and concentration for steady flow of nanofluid can be represented as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\sqrt{2}\nu\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{k'}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]-\frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y}+ \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-k_0C \tag{4}$$

The corresponding boundary conditions are:

$$u=U_w(x)=ax,\qquad T=T_m,\ k\left(\frac{\partial T}{\partial y}\right)=\rho\left(\lambda+c_s\left(T_m-T_s\right)\right)v(x,0),\ C=C_w\ \ at\ y=0,$$

$$u=0,\ T\to T_\infty\qquad C\to C_\infty\qquad \text{as}\qquad y\to\infty \tag{5}$$

where u, v are the velocity components along the x and y axes, ρ is density of the nanofluid, ν is the kinematic viscosity, B_o is the induced magnetic field, k' is the permeability of porous medium, τ is the ratio between the effective heat capacity of the nanoparticle material and the fluid, Γ is time constant, T is the nanofluid temperature, C is the volumetric volume expansion coefficient, T_w is the temperature of the nanofluid near wall, T_∞ is the free stream temperature of the nanofluid, k is the thermal conductivity, T_m is the melting temperature, T_s is the temperature of solid surface, λ is the latent heat of the fluid, c_s is the heat capacity of solid surface, U_w is the stretching sheet velocity, a is the stretching rate being a positive constant, c_p is the specific heat at constant pressure, q_r is the radiative heat flux, Q' is the heat generation coefficient, D_B is the

Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient, k_o is the chemical reaction coefficient.

Using Rosseland approximation for radiation, the radiative heat flux is simplified as,

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* is the Stephen Boltzmann constant and k^* is the mean absorption coefficient.

It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences within the flow are sufficiently small, then equation (6) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

Substituting (6) and (7) in (3), we have

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{16\sigma^*}{3k^*}\frac{T_\infty^3}{(\rho c)_f}\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q'}{(\rho c)_f}(T - T_\infty) \tag{8}$$

The governing equations can be reduced to ordinary differential equations, using the following similarity transformation,

$$\psi = (a\nu)^{\frac{1}{2}} x f(\eta), \quad \eta = \left(\frac{a}{\nu}\right)^{\frac{1}{2}} y, \quad \theta(\eta) = \frac{T - T_m}{T_\infty - T_m}, \quad \phi(\eta) = \frac{C - C_w}{C_\infty - C_w}, \tag{9}$$

The stream function ψ is defined such that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$

With the help of transformations, equation (1) is clearly satisfied, and equations (2), (4) and (8) along with boundary condition (5) take into the following form

$$f'''(\eta)+f(\eta)f''(\eta)-\left(f'(\eta)\right)^2+\lambda f''(\eta)f'''(\eta)-(M+K)f'=0 \tag{10}$$

$$\left(1+\frac{4}{3}R\right)\theta''(\eta)+\Pr f(\eta)\theta'(\eta)+\Pr Nb\theta'(\eta)\phi'(\eta)+\Pr Nt\left(\theta'(\eta)\right)^2+\Pr Ecf''^2+\Pr Q\theta(\eta)=0 \tag{11}$$

$$\phi''(\eta)+Lef(\eta)\phi'(\eta)+\frac{Nt}{Nb}\theta''(\eta)-\gamma\phi(\eta)=0 \tag{12}$$

The corresponding boundary conditions will take the form

$$f'(0)=1.0,\ \Pr f(0)+Me\theta'(0)=0,\ \theta(0)=0,\ \phi(0)=0,$$

$$f'(\infty)=0.0,\quad \theta(\infty)=1.0,\quad \phi(\infty)=1.0. \tag{13}$$

$M=\frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter; $K=\frac{\nu}{k'a}$ is the permeability parameter; $\lambda=\Gamma x\left(\frac{2a^3}{\nu}\right)^{\frac{1}{2}}$ is the non-Newtonian Williamson parameter; $\alpha_m=\frac{k}{(\rho c)_f}$ is thermal diffusivity of nanofluid;

$\Pr=\frac{\nu}{\alpha_m}$ is Prandtl number; $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is ratio between the effective heat capacity of the nanoparticle material and the fluid; $Q=\frac{Q'}{a\rho c_p}$ is the heat generation parameter; $Ec=\frac{U_w^2}{c_p\left(T_\infty-T_m\right)}$ is the Eckert number; $R=\frac{4\sigma^* T_\infty^3}{kk^*}$ is the radiation parameter; $Nb=\frac{\tau D_B\left(C_\infty-C_w\right)}{\nu}$ is Brownian motion parameter; $Nt=\frac{\tau D_T\left(T_\infty-T_m\right)}{\nu T_\infty}$ is the themophoresis parameter; $Le=\frac{\nu}{D_B}$ is the Lewis number; $Kr=\frac{k_o U_w\left(C_\infty-C_w\right)}{\nu}$ is the chemical reaction parameter; $Me=\frac{c_f\left(T_\infty-T_m\right)}{\lambda+c_s\left(T_m-T_s\right)}$ is the dimensionless parameter;

The physical quantities of interest like skin-friction coefficient $\left(C_f\right)$, local Nusselt number $\left(Nu_x\right)$ and local Sherwood number $\left(Sh_x\right)$ are defined as

$$C_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{x q_w}{k(T_\infty - T_m)} \quad \text{and} \quad Sh_x = \frac{x q_m}{D_B (C_\infty - C_m)} \tag{14}$$

where the shear stress (τ_w), surface heat flux (q_w) and surface mass flux (q_m) are given by

$$\tau_w = \mu \left[\frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}} \left(\frac{\partial u}{\partial y} \right)^2 \right], \quad q_w = -k \frac{\partial T}{\partial y}, \quad q_m = -D_B \frac{\partial C}{\partial y} \quad \text{at } y = 0$$

Using the non-dimensional variables, we obtain

$$C_f \left(\mathrm{Re}_x\right)^{-\frac{1}{2}} = f''(0) + \frac{\lambda}{2} f''(0)^2, \quad Nu_x \left(\mathrm{Re}_x\right)^{-\frac{1}{2}} = -\theta'(0) \quad \text{and} \quad Sh_x \left(\mathrm{Re}_x\right)^{-\frac{1}{2}} = -\phi'(0)$$

where $\mathrm{Re}_x = \frac{x U_w(x)}{\nu}$ is the local Reynold's number.

Results and discussion

We reveal the results to keep up the influence of several non-dimensional parameters such as melting parameter, magnetic field parameter and other parameters on the three usual profiles (velocity, temperature and concentration). Also we examined the same parameters on skin friction coefficient, heat transfer rate and mass transfer rate with the aid of table. In this paper, we have chosen the non-dimensional parameter values as $M = 0.5$, $Me = 0.5$, $K = 0.5$, $\lambda = 0.2$, $R = 0.01$, $Pr = 2$, $Nb = 0.1$, $Nt = 0.1$, $Le = 10.0$, $Ec = 0.01$, $Q = 0.01$ *and* $Kr = 0.05$. These values are maintained as invariable in this study unless the varied parameters as depicted in the figures.

In figures 1- 20, we presented the highlights of the effects of the governing parameters on the nanofluid velocity, temperature and concentration profiles on the plate surface.

Figures 1 -3 depict the effects of magnetic field parameter on velocity, temperature and concentration profiles, respectively. it is observed that all the fields are decreases significantly with increasing magnetic field parameter. This would happen because the application of a transverse magnetic field sets up the Lorentz force, which retards the nanofluid velocity.

The effects of different values of permeability parameter on velocity, temperature and concentration fields are presented in Figs. 4 – 6. It is obvious that the presence of porous medium causes higher restriction to the fluid flow, which in turn slows its motion. Therefore, with increasing permeability parameter, the resistance to the fluid motion increases and hence velocity decreases. Temperature and concentration fields are also decreases with increasing permeability parameter. Figs. 7 – 8 explain the effect of melting parameter on velocity, temperature and concentration fields. It is noticed that for increasing values of melting parameter values, the velocity and the boundary layer thickness increase and decrease the temperature and concentration profiles. Reason for this behavior is an increase in melting parameter will increase the intensity of melting, which acts as blowing boundary condition at the surface and hence tends to thicken the boundary layer.

The variation of velocity, temperature and concentration distributions with non-Newtonian Williamson parameter is studied in Figs. 10 – 12. From these figures we observed that the nanofluid velocity, temperature and concentration decreases with increase in non-Newtonian Williamson parameter.

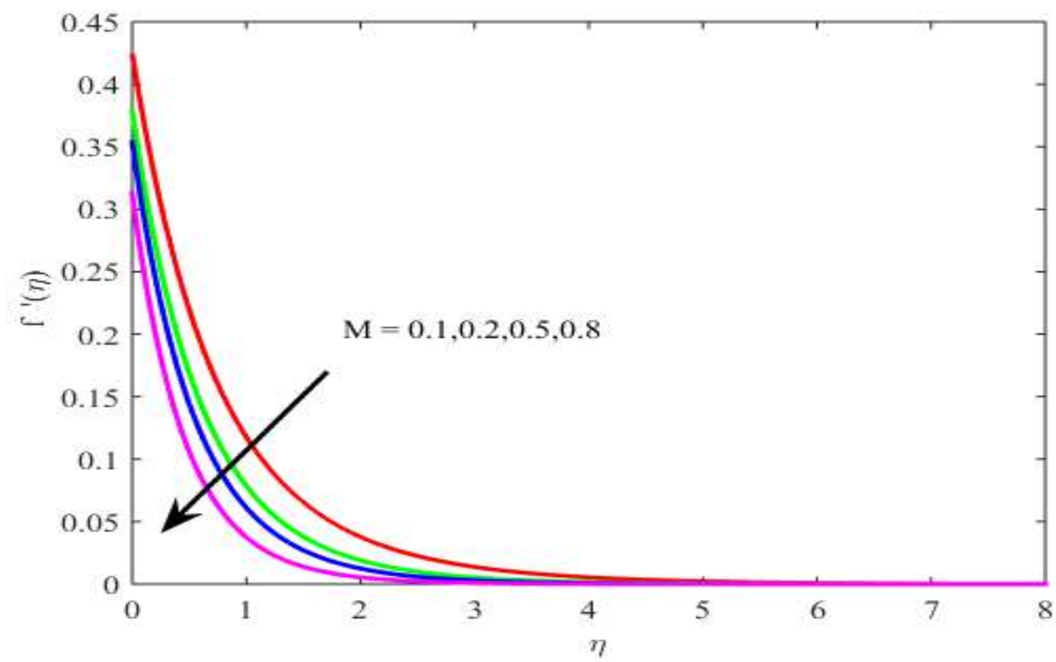

Fig.2 Influence of M on velocity profile.

Fig.2 exhibits the decreasing nature of the velocity profile $f'(\eta)$ and also the boundary layer thickness for higher values of M . This indicates that the increase in M helps to thin of the boundary layer. The velocity profiles exponentially reduce to zero at shorter distances from the sheet for growing values of M .

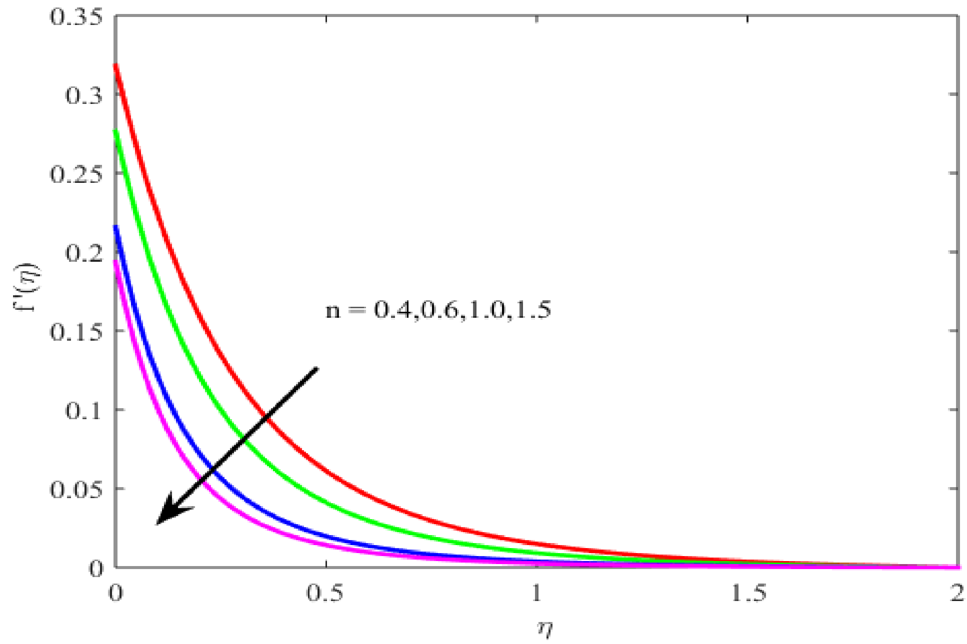

Fig.3 Influence of n on velocity profile.

Fig.3 shows the impact of the curvature parameter on non-dimensional velocity distribution $f'(\eta)$. A rise in the curvature parameter results , decrease in the nondimensional velocity. Resistance force is created by the magnetic field on the fluid in the boundary layer. This force causes restriction to the motion of the fluid. So the magnetic parameter reduces the dimensionless velocity.

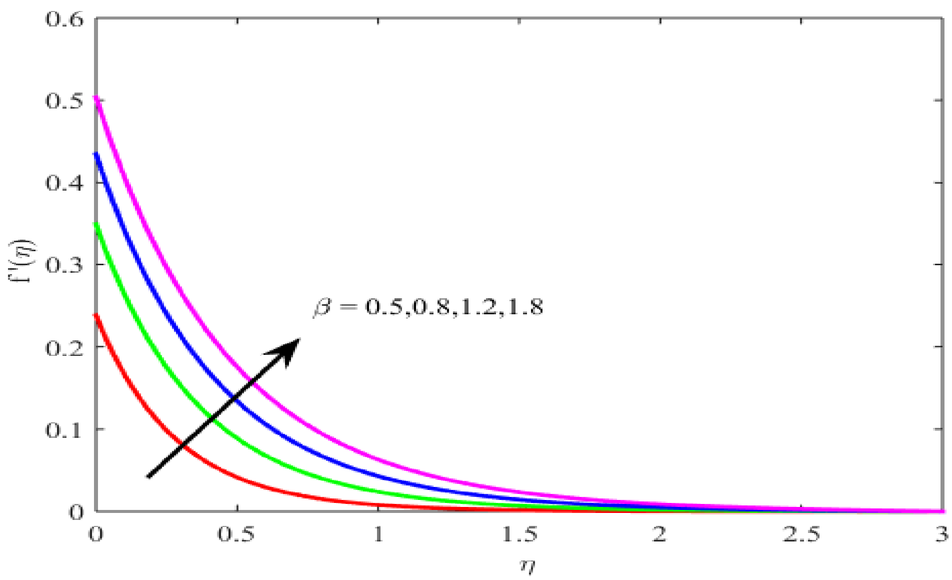

Fig.4 Influence of β on velocity profile.

Fig.4 explicates the increasing nature of velocity profile for rising values of Casson fluid parameter.

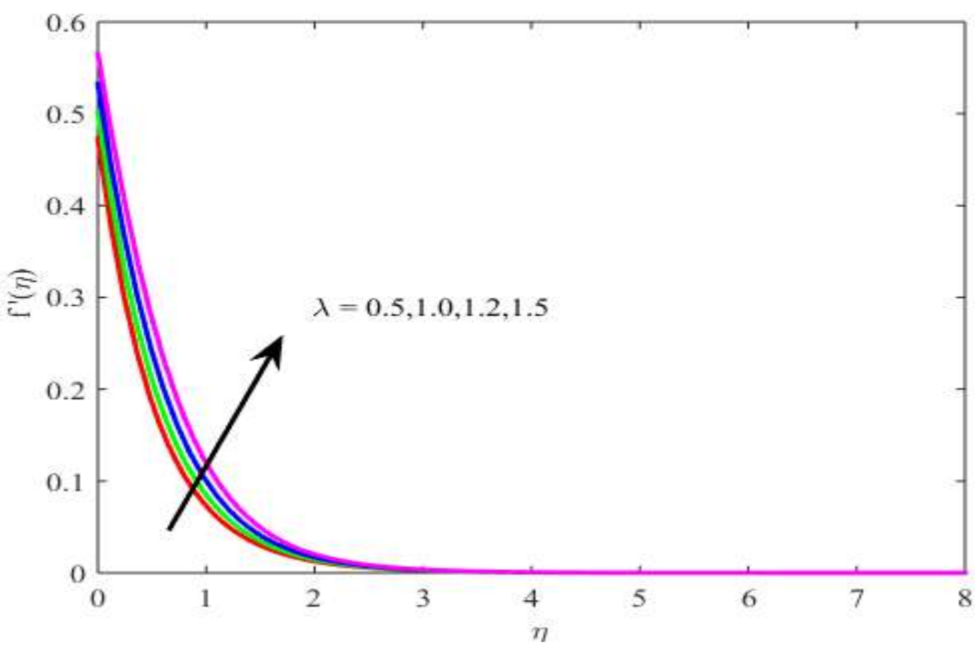

Fig.5 Influence of λ on velocity profile.

Fig.5 illuminates the effect the Wiesenberger number λ on velocity profile. It depicts the velocity profile increased with the increment of λ .

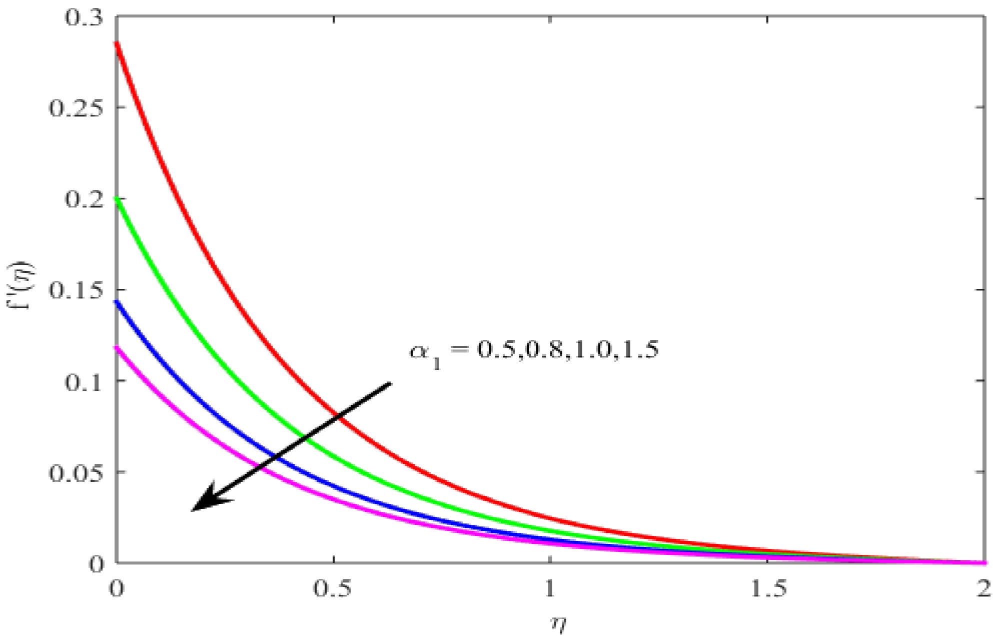

Fig.6 Influence of α_1 on velocity profile.

Fig.6 explains the influence of first-order velocity slip parameter on the dimensionless velocity profile $f'(\eta)$. The dimensionless velocity profile $f'(\eta)$ decreases with increasing values of the first-order velocity slip parameter α_1.

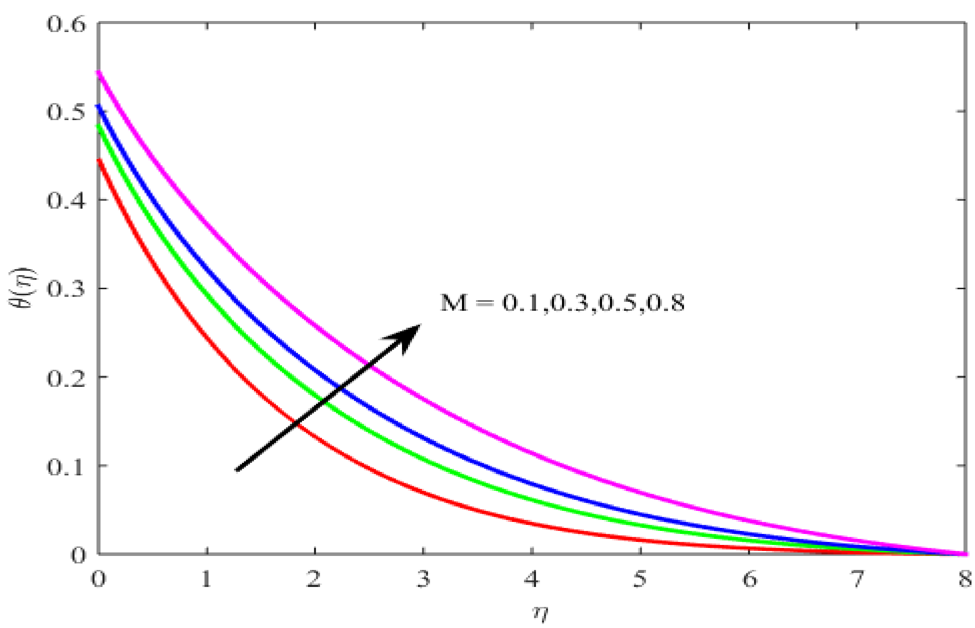

Fig.7 Influence of M on temperature profile.

Fig.7 represents the impact of the magnetic parameter on energy distribution. The effect of magnetic field reduces the fluid velocity whereas it intensifies thermal boundary layer thickness. Thermal energy is defined as an additional work done required for dragging the fluid under the influence of the magnetic field. Thermal energy heats up the conducting fluid and upgrades the temperature profile. Thus, the magnetic field in the flow regime intensifies the thermal boundary layer thickness.

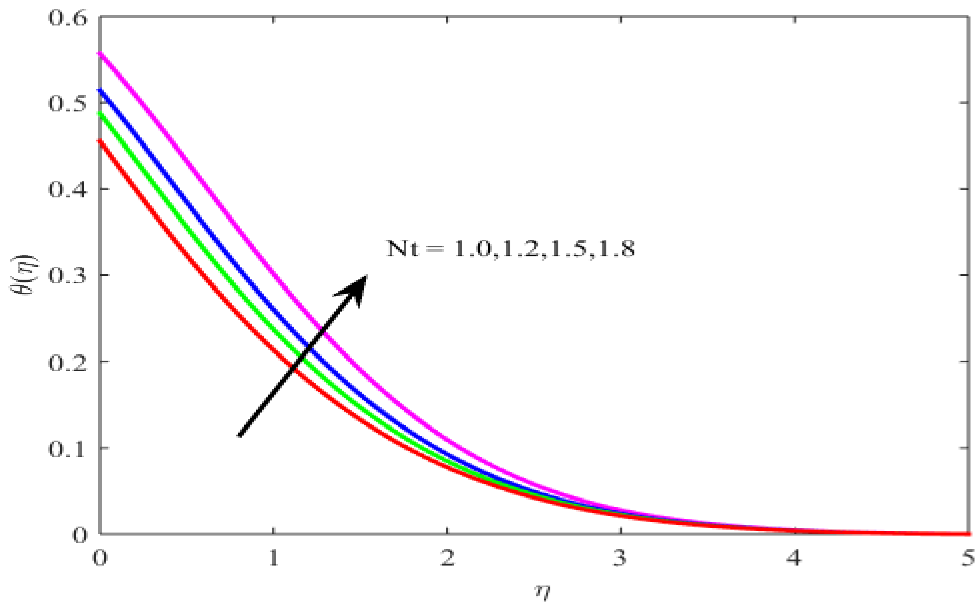

Fig.8 Influence of Nt on temperature profile.

Fig.8 exhibits the influence of the thermophoresis parameter on temperature profile. The energy distribution grows with increment in the values of the thermophoresis parameter.

Conclusion

In the present paper, we have studied the impacts of the thermal radiation, heat source and melting parameter on steady MHD boundary layer of nanofluid under influence of chemical reaction embedded in porous medium. Using the similarity variables, the governing non-linear partial differential equations are transformed into a system of coupled non-linear ODE's and solved numerically by using Runge-Kutta – Fehlberg method with shooting technique. Velocity and temperature increase with increase in heat generation parameter or viscous dissipation. velocity profile, temperature profile and concentration profile decreases for increasing the values of permeability parameter or magnetic field parameter. both local Nusselt number and local Sherwood number increase with Lewis number. The heat source parameter and Eckert number increases the heat transfer rate, but decreases the mass transfer rate. The skin-friction factor, Nusselt number and Sherwood number decreases with increase in the melting parameter, whereas the effect is opposite for Prandtl number.

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